

Annuity

In [investment](#), an **annuity** is a series of payments made at equal intervals based on a contract with a lump sum of money.^[1] Insurance companies are common annuity providers and are used by clients for things like retirement or death benefits.^[2] Examples of annuities are regular deposits to a [savings account](#), monthly [home mortgage](#) payments, monthly [insurance](#) payments and [pension](#) payments. Annuities can be classified by the frequency of payment dates. The payments (deposits) may be made weekly, monthly, quarterly, yearly, or at any other regular interval of time. Annuities may be calculated by [mathematical functions](#) known as "annuity functions".

An annuity which provides for payments for the remainder of a person's lifetime is a [life annuity](#). An annuity which continues indefinitely is a [perpetuity](#).

Types

Annuities may be classified in several ways.

Timing of payments

Payments in an [annuity-immediate](#) are made at the end of each period, so interest accrues before the first payment. Payments in an [annuity-due](#) are made at the beginning of each period, so a payment is made at inception.

Contingency of payments

An annuity that pays over a fixed period is an annuity certain (also called a guaranteed annuity). An annuity paid only if stated conditions are met is a contingent annuity. A common example is a [life annuity](#), which is contingent on survival. Certain-and-life annuities pay for a minimum number of years and thereafter only while the annuitant is alive.

Variability of payments

- **Fixed annuities** – Annuities with fixed payments. When provided by an insurer, the contract credits a fixed rate specified in the contract. In the United States, fixed annuities are regulated by state

insurance commissioners and are not securities regulated by the Securities and Exchange Commission.^[3]

- **Variable annuities** – In the United States, these are registered with the [Securities and Exchange Commission](#) and allow investment in separate accounts holding underlying funds. Contracts often include guaranteed death benefits or lifetime withdrawal benefits.
- **Equity-indexed annuities** – Annuities with crediting linked to an external index. Contracts set a floor (often 0%) and a cap; the index performance determines the credited rate within that range.

Deferral of payments

A deferred annuity begins payments after a deferral period. An immediate annuity begins payments without a deferral period, typically at purchase.^{[4][5]}

Valuation

Valuation of an annuity calculates the [present value](#) of its future payments. Key concepts include the [time value of money](#), the per-period [interest rate](#), and [future value](#).^[6]

Annuity-certain

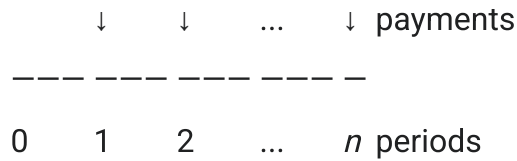
If the number of payments is known in advance, the contract is an annuity certain (also called a guaranteed annuity). Valuation uses the formulas below, which depend on the timing of payments.

Annuity-immediate

If payments are made at the end of each period (so interest accrues before payment), the annuity is an annuity-immediate (ordinary annuity). Mortgage payments are annuity-immediate, interest accrues before it is paid.

Annuity due

Annuity due refers to a series of equal payments made at the same interval at the beginning of each period. Periods can be monthly, quarterly, semi-annually, annually, or any other defined period. Examples of annuity due payments include rentals, leases, and insurance payments, which are made to cover services provided in the period following the payment.



For an annuity-immediate, the present value is the value of a stream of payments, discounted by the interest rate to account for the fact that payments are being made at various moments in the future. The present value is given in [actuarial notation](#) by:

$$a_{\overline{n}|i} = \frac{1 - (1 + i)^{-n}}{i},$$

where n is the number of terms and i is the per period interest rate. Present value is linear in the amount of payments, therefore the present value for payments, or *rent* R is:

$$PV(i, n, R) = R \times a_{\overline{n}|i}.$$

In practice, often loans are stated per annum while interest is compounded and payments are made monthly. In this case, the interest I is stated as a [nominal interest rate](#), and $i = I/12$.

The future value is the accumulated amount, including payments and interest, of a stream of payments made to an interest-bearing account. For an annuity-immediate, it is the value immediately after the n -th payment. The future value is given by:

$$s_{\overline{n}|i} = \frac{(1 + i)^n - 1}{i},$$

where n is the number of terms and i is the per period interest rate. Future value is linear in the amount of payments, therefore the future value for payments, or *rent* R is:

$$FV(i, n, R) = R \times s_{\overline{n}|i}$$

Example: The present value of a 5-year annuity with a nominal annual interest rate of 12% and monthly payments of \$100 is:

$$PV\left(\frac{0.12}{12}, 5 \times 12, \$100\right) = \$100 \times a_{\overline{60}|0.01} = \$4,495.50$$

The rent is understood as either the amount paid at the end of each period in return for an amount PV borrowed at time zero, the *principal* of the loan, or the amount paid out by an interest-bearing account at the end of each period when the amount PV is invested at time zero, and the account becomes zero with the n -th withdrawal.

Future and present values are related since:

$$s_{\overline{n}|i} = (1+i)^n \times a_{\overline{n}|i}$$

and

$$\frac{1}{a_{\overline{n}|i}} - \frac{1}{s_{\overline{n}|i}} = i$$

Proof of annuity-immediate formula

To calculate present value, the k -th payment must be discounted to the present by dividing by the interest, compounded by k terms. Hence the contribution of the k -th payment R would be $\frac{R}{(1+i)^k}$.

Just considering R to be 1, then:

$$\begin{aligned} a_{\overline{n}|i} &= \sum_{k=1}^n \frac{1}{(1+i)^k} = \frac{1}{1+i} \sum_{k=0}^{n-1} \left(\frac{1}{1+i} \right)^k \\ &= \frac{1}{1+i} \left(\frac{1 - (1+i)^{-n}}{1 - (1+i)^{-1}} \right) \quad \text{by using the equation for the sum of a geometric series} \\ &= \frac{1 - (1+i)^{-n}}{1+i-1} \\ &= \frac{1 - \left(\frac{1}{1+i} \right)^n}{i}, \end{aligned}$$

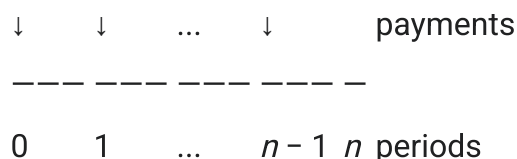
which gives us the result as required.

Similarly, we can prove the formula for the future value. The payment made at the end of the last year would accumulate no interest and the payment made at the end of the first year would accumulate interest for a total of $(n-1)$ years. Therefore,

$$s_{\overline{n}|i} = 1 + (1+i) + (1+i)^2 + \cdots + (1+i)^{n-1} = (1+i)^n a_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}.$$

Annuity-due

An *annuity-due* is an annuity whose payments are made at the beginning of each period.^[7] Deposits in savings, rent or lease payments, and insurance premiums are examples of annuities due.



Each annuity payment is allowed to compound for one extra period. Thus, the present and future values of an annuity-due can be calculated.

$$\ddot{a}_{\overline{n}|i} = (1+i) \times a_{\overline{n}|i} = \frac{1 - (1+i)^{-n}}{d},$$

$$\ddot{s}_{\overline{n}|i} = (1+i) \times s_{\overline{n}|i} = \frac{(1+i)^n - 1}{d},$$

where n is the number of terms, i is the per-term interest rate, and d is the [effective rate of discount](#) given by $d = \frac{i}{i+1}$.

The future and present values for annuities due are related since:

$$\ddot{s}_{\overline{n}|i} = (1+i)^n \times \ddot{a}_{\overline{n}|i},$$

$$\frac{1}{\ddot{a}_{\overline{n}|i}} - \frac{1}{\ddot{s}_{\overline{n}|i}} = d.$$

Example: The final value of a 7-year annuity-due with a nominal annual interest rate of 9% and monthly payments of \$100 can be calculated by:

$$FV_{\text{due}} \left(\frac{0.09}{12}, 7 \times 12, \$100 \right) = \$100 \times \ddot{s}_{\overline{84}|0.0075} = \$11,730.01.$$

In Excel, the PV and FV functions take on optional fifth argument which selects from annuity-immediate or annuity-due.

An annuity-due with n payments is the sum of one annuity payment now and an ordinary annuity with one payment less, and also equal, with a time shift, to an ordinary annuity. Thus we have:

$$\ddot{a}_{\overline{n}|i} = a_{\overline{n}|i}(1+i) = a_{\overline{n-1}|i} + 1. \text{ The value at the time of the first of } n \text{ payments of } 1.$$

$$\ddot{s}_{\overline{n}|i} = s_{\overline{n}|i}(1+i) = s_{\overline{n+1}|i} - 1. \text{ The value one period after the time of the last of } n \text{ payments of } 1.$$

Perpetuity

A *perpetuity* is an annuity for which the payments continue forever. Observe that

$$\lim_{n \rightarrow \infty} PV(i, n, R) = \lim_{n \rightarrow \infty} R \times a_{\overline{n}|i} = \lim_{n \rightarrow \infty} R \times \frac{1 - (1+i)^{-n}}{i} = \frac{R}{i}.$$

Therefore a [perpetuity](#) has a finite present value when there is a non-zero discount rate. The formulae for a perpetuity are

$$a_{\overline{\infty}|i} = \frac{1}{i} \text{ and } \ddot{a}_{\overline{\infty}|i} = \frac{1}{d},$$

where i is the interest rate and $d = \frac{i}{1+i}$ is the effective discount rate.

Life annuities

Valuation of [life annuities](#) may be performed by calculating the [actuarial present value](#) of the future life contingent payments. [Life tables](#) are used to calculate the [probability](#) that the annuitant lives to each future payment period. Valuation of life annuities also depends on the timing of payments just as with annuities certain, however life annuities may not be calculated with similar formulas because actuarial present value accounts for the probability of death at each age.

Amortization calculations

If an annuity is used to repay a debt with end-of-period payments (an annuity-immediate), and with principal P , per-period rate i , and level payment R , the outstanding balance after n payments is

$$B_n = (1+i)^n P - R \frac{(1+i)^n - 1}{i} = \frac{R}{i} - (1+i)^n \left(\frac{R}{i} - P \right).$$

Equivalently (prospective view), if the total number of payments is N , after n payments the outstanding balance is the present value of the remaining $N - n$ payments:

$$B_n = R a_{\overline{N-n}|i} = R \frac{1 - (1+i)^{-(N-n)}}{i}.$$

For an annuity-due (payments at the beginning of each period), multiply annuity-immediate factors by $(1+i)$:

$$B_n^{(\text{due})} = R \ddot{a}_{\overline{N-n}|i}, \quad \ddot{a}_{\overline{m}|i} = (1+i) a_{\overline{m}|i}.$$

Example (check). Let $P = 1,000$, $i = 0.10$, $n = 3$ (annuity-immediate). The level payment is

$$R = \frac{Pi}{1 - (1+i)^{-n}} \approx 402.11.$$

After one payment,

$$B_1 = 1,000 \cdot 1.1 - 402.11 \cdot \frac{1.1 - 1}{0.10} \approx 697.89 = R a_{\overline{2}|0.10}.$$

See also [Fixed rate mortgage](#).

Example calculations

This section gives worked examples for finding the periodic payment R from either the present value A or the accumulated value S for an annuity-due. We use the effective per-period rate $i = j/m$ and the number of payments n throughout.

Formula for finding the periodic payment R , given A (present value of an annuity-due):

$$R = \frac{A}{\left(\frac{1-(1+i)^{-n}}{i} \right) (1+i)} \quad \text{where } i = \frac{j}{m}.$$

(An annuity-due pays at the beginning of each period. For an ordinary annuity (annuity-immediate), omit the final $(1+i)$ factor in the denominator.)

Examples. Values are rounded to at most two decimal places.

Example 1. Present value to payment (annuity-due). Let $i = 0.15$, $n = 3$. The annuity-due factor is

$$\ddot{a}_3 = \left(\frac{1 - (1 + 0.15)^{-3}}{0.15} \right) (1 + 0.15) = 2.63.$$

Hence

$$R = \frac{70,000}{2.63} \approx \$26,659.47.$$

Cross-check (ordinary annuity first): the present-value factor is

$$a_3 = \frac{1 - (1 + 0.15)^{-3}}{0.15} = 2.28.$$

Multiplying by $(1+i)$ gives

$$\ddot{a}_3 = a_3(1 + 0.15) = 2.63.$$

Hence $R \approx \$26,659.47$.

Example 2. Present value to payment (annuity-due). \$250,700 payable quarterly for 8 years at 5% compounded quarterly. Let $i = 0.05/4 = 0.0125$, $n = 8 \times 4 = 32$. Then

$$\ddot{a}_{32} = \left(\frac{1 - (1 + 0.0125)^{-32}}{0.0125} \right) (1 + 0.0125) = 26.57.$$

Therefore

$$R = \frac{250,700}{26.57} \approx \$9,435.71.$$

Formula for finding the periodic payment R , given S (accumulated value of an annuity-due):

$$R = \frac{S}{\left(\frac{(1+i)^{n+1}-1}{i} \right) - 1} \quad \text{where } i = \frac{j}{m}.$$

Example 3. Accumulated value to payment (annuity-due). \$55,000 payable monthly for 3 years at 15% compounded monthly. Let $i = 0.15/12 = 0.0125$, $n = 3 \times 12 = 36$. Then

$$\ddot{s}_{36} = \left(\frac{(1 + 0.0125)^{37} - 1}{0.0125} \right) - 1 = 45.68.$$

Therefore

$$R = \frac{55,000}{45.68} \approx \$1,204.04.$$

Example 4. Accumulated value to payment (annuity-due). \$1,600,000 payable annually for 3 years at 9% compounded annually. Let $i = 0.09$, $n = 3$. Then

$$\ddot{s}_3 = \left(\frac{(1 + 0.09)^4 - 1}{0.09} \right) - 1 = 3.57.$$

Therefore

$$R = \frac{1,600,000}{3.57} \approx \$447,786.80.$$

Legal regimes

- [Annuities under American law](#)
- [Annuities under European law](#)
- [Annuities under Swiss law](#)

See also

- [Amortization calculator](#)
- [Fixed rate mortgage](#)
- [Life annuity](#)
- [Perpetuity](#)
- [Time value of money](#)

References

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